# Optical Modulation via QCSE

A plain-language report on the quantum confined stark effect and its use in optical modulators

> By Oisín Davey Based on my work at Tyndall National Institute 3 rd of August, 2023

# Contents



### <span id="page-2-0"></span>1 Note to the reader

In this report I attempt to explain the area of my work as part of the Tyndall fellowship verbosely, in such a way that would've helped myself in the past. The jargon/terminology/basic assumptions of industry photonics research stifled my progress more than anything else during my work. I will appeal to ideas in electronics in order to construct a frame of reference for photonics. Ideally, a first-year physics undergraduate with a basic grasp of quantum mechanics should be able to understand this report. This report is produced voluntarily i.e., I was not paid to produce this document and am not submitting this document for a grade in my degree.

The work referenced in this report was produced under supervision of Professor Frank Peters, with guidance from Eric Murphy and Dr. Xing Dai, as part of the Integrated Photonics Group.

# <span id="page-2-1"></span>2 What's an Optical modulator?

#### <span id="page-2-2"></span>2.1 Modulation: what? why?

Put simply, photonics serves as an alternative to electronics, where computations are done with light rather than with electrons. There are analogues between the two, namely the Photonic Integrated Circuit (PIC) as opposed to the typical electric Integrated Circuit (IC). Rather than integrating the circuitry into silicon, a PIC integrates photonics into other semiconductors like Indium Phosphide, allowing for lasers to operate in the same material.

Where electronics have resistors, capacitors, and diodes as circuit elements, photonics have isolators, modulators, cavities, etc. Modulators have lots of different principles of operation, and they can modulate many different properties. An electromagnetic wave can have, for example, its amplitude or frequency modulated, as in AM/FM radio.

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Figure 1: Above: Amplitude modulation, Below: Frequency modulation

For an electro-absorption modulator (EAM), the absorption spectrum of a medium is what gets modulated, essentially turning on/off a portion of the spectrum through application of an electric field. In this sense, it may be helpful to think of EAMs as the optical counterpart to the electrical transistor, to understand why it might be so useful.

Suppose you have a laser of wavelength  $\lambda$ , passing through an EAM designed to operate at  $\lambda$ . If no voltage is applied accross the EAM, the laser will be largely unaffected. However, if the voltage is sufficiently raised, the  $\lambda$  wavelength light will be absorbed by the EAM.

It's like a lightswitch, but little.

#### <span id="page-3-0"></span>2.2 Semiconductor photonics: what?

In a given medium of interest, there are two main energy bands in which an electron may reside: the valence band and the conduction band. An electron in the valence band is basically bound to an atom, whereas an electron in the conduction band has more energy and is free to move around the semiconductor.

When an electron jumps to the conduction band through absorption of a photon, it leaves behind a hole in the valence band. This hole is a quasi-particle<sup>[1](#page-3-1)</sup> with positive mass and charge, and may also be governed by the schrödinger equation.



Figure 2: An electron jumps between bands via photon absorption, creating a hole

When a laser shines on the medium, photons may be absorbed, but when is this possible, and how likely is it? The first criterion is that the photon has enough energy to promote an electron from the valence band to the conduction band. Below this energy, the transition isn't permitted. Recall that the energy of a photon is given by  $\hbar\omega = hf = \frac{hc}{\lambda}$ . It's required then, that  $\hbar\omega \geq E_c - E_v$ , in order for the medium to absorb the photon and promote the electron. This quantity  $E_c - E_v$  is known as the band-gap of the material.

<span id="page-3-1"></span><sup>&</sup>lt;sup>1</sup>Important subtlety: The electron in question is also a quasi particle, and does not technically refer to the fundamental particle of the same name. Nonetheless this quasi particle may be governed by the schrödinger equation within the kronig-penny model. These particles have tensor mass, rather than scalar mass.

Not all absorptions are equally likely. When autumn light shines on the leaf of a decidu-ous tree<sup>[2](#page-4-0)</sup>, the oranges and reds aren't absorbed very much, and reflect back into your eyes. How do we account for this?

Within convenient models, Fermi's golden rule implies that the probability of a transition is in proportion with the overlap between the electron wavefunction and hole wavefunction. Suppose an electron gets promoted to the conduction band and leaves behind a hole, the wavefunctions of the electron and the hole are denoted by  $\psi_c$  and  $\psi_v$ , respectively. This promotion is only possible if  $\psi_e$  looks like  $\psi_h$ .



Figure 3: Left: Good overlap in wavefunctions, Right: Bad overlap in wavefunction

To be more precise, the probability of a  $\hbar\omega$  absorption obeys  $\alpha(\hbar\omega) \sim \sum$ c,v  $|\langle \psi_c, \psi_v \rangle|^2$ . Here we're running over all possible electron-hole pairs  $c, v$ , and totalling their contribution to the absorption probability.

For standard semiconductor materials, the absorption spectrum looks like the below graph. As you can see there is no absorption below the bandgap, as expected. Beyond that point, the absorption increases with the square root of the photon energy.



Figure 4: Bulk absorption for generic semiconductor

<span id="page-4-0"></span><sup>2</sup>Not a very effective semiconductor

#### <span id="page-5-0"></span>2.3 EAMs: how?

There are two major ways to operate an EAM: The Franz-Keldysh effect  $\&$  The quantum-confined Stark effect (QCSE). Both methods operate on the same guiding principal: use an electric field to control overlaps.

Recall the time independent schrödinger equation: −  $\hbar^2$ 2m  $\partial^2\psi$  $\frac{\partial^2 \psi}{\partial z^2} + V \psi = E \psi$ . For EAMs you only need to be concerned with one dimension. Let's just consider a static electric field, so  $V = kz$  for some constant k. This field allows wavefunctions given by the following expression:

$$
\psi = c \cdot \text{Ai}\left(\left[\frac{2m}{\hbar^2}\right]^{\frac{1}{3}} \left[k^{\frac{1}{3}}z - Ek^{-\frac{2}{3}}\right]\right)
$$

Here  $Ai(x)$  is the Airy function of the first kind. Feel free to verify this but what's more important is what this wave looks like. Below you can see a pair of wavefunction solutions for a certain electron-hole pair. See how they look sinusoidal until  $E = V$  and then they have a long tail off. First thing to note is that the transition energy  $E_c - E_v$  is actually less than the material bandgap. Whats-more, these tails overlap! This overlap allows the small possibility of this previously forbidden transition.



Figure 5: Airy wavefunctions

What does this mean for absorption?



Figure 6: The Franz-Keldysh effect. Absorption without field (grey) and with field (sparkly)

Now there is absorption happening below the band-edge, where it used to be strictly forbidden! This is the foundational idea for Franz-Keldysh EAMs: The electric field controls whether light just a little redder than the bandgap is absorbed or allowed to pass through.

EAMs based on this principal have been made and circulated successfully, but this is not where I've focused my work with Tyndall. Why is this?<sup>[3](#page-6-2)</sup> QCSE is more interesting and more efficient<sup>[4](#page-6-3)</sup>.

# <span id="page-6-0"></span>3 What's the quantum confined stark effect?

For starters, let's focus on "stark effect". The stark effect used to mean something, specifically involving observations of the changes in spectral lines in the presence of an electric field. Those words are certainly a lot like what we're talking about, but this interpretation would imply that we should call it the Franz-Keldysh-Stark effect. For our purposes "stark effect" can be interpretted as a buzzword to indicate applicability in EAMs.

It comes then, to discuss the relevance of "quantum confinement", and to do this we first have to talk about yet another quasi-particle: the exciton.

#### <span id="page-6-1"></span>3.1 Excitons: huh?

Hydrogen (H): a universal fan favorite element, the O.G. of the periodic table. Consider H in terms of the electro-magnetic force; you have two masses of equal and opposite charge bound together. The energy levels of the electron are quantised and the wavefunction is governed by the spherically symmetric schrödinger equation.



Figure 7: My mental picture of an exciton

Any particle that can be described this way may be called *hydrogenic*, for example consider positronium: the short lived bound pair of an electron and a positron. Both electrons and positrons are well-observed fundamental particles, so it's not too controversial to say that positronium is a *real particle*. This brings us to the exciton: a hydrogenically bound electron-hole pair.

<span id="page-6-2"></span><sup>3</sup>Because the people funding my research did not care for it

<span id="page-6-3"></span><sup>4</sup>By an admittedly arbitrary figure of metric

A hole has postive mass, equal in magnitude to that of the electron, so this is perfectly reasonable. It may bother you to let the exciton into your life, because the hole is only tentatively a particle. An exciton could be described fairly accurately as an electron bound to where it isn't.

The lifespans of excitons are incredibly short at room temperature at standard conditions, as you may well imagine. This is why their effects don't appear in the bulk semiconductor absorption spectrum. Every time an electron gets promoted and leaves a hole behind, it may very briefly form an exciton. For our nefarious scheme, we would like to extend this lifespan. More effects in the absorption means more effects to exploit for modulation; this is where quantum confinement comes into play.

#### <span id="page-7-0"></span>3.2 Quantum confinement: how?

If we can restrict the range of motion of the electrons and holes, we can give the excitons a fighting chance. The key idea? Put them into quantum wells.

A finite quantum well may be constructed by adding in a new material (material B) with lower  $E_c$  and higher  $E_v$  than the old material (material A), see what this does to the potential before applying a field:



Figure 8: Quantum well structure with an electron and a hole trapped in wells

So now the electron and hole are confined in quantum wells, allowing the exciton to sustain its existence at room temperature. Before we investigate what effect excitons have on the spectrum, we must first consider what quantum well solutions do to the spectrum.

We still have the standard bulk behavior after the bandgap of material  $A$  (see figure 4), but this won't be of much interest, so lets zoom in to the bandgap of material B. This is to say we'll only care about the bound solutions of the finite quantum well (SQW). Hopefully you'll be familiar with the solutions to SQWs shown below.



Figure 9: The first three wavefunctions of an SQW

From Fermi's rule, we only get transitions between  $n = 1$  electrons and  $n = 1$  holes,  $n = 2$  electrons and  $n = 2$  holes, and so on (This is known as a selection rule in chemistry). This means that we get a quantised step-like version of the old bulk behavior.



Figure 10: Absorption spectrum of SQW (no temperature, no excitons)

So what effect do excitons have on this spectrum? Less energy is needed to have one exciton than the free electron and hole, because of this binding. This creates peaks in absorption slightly redshifted from the steps. The steps and the peaks get smeared and blurred due to thermal effects and defects in the semiconductor.



Figure 11: Realistic SQW absorption spectrum, with two major exciton peaks

There are usually only two visible excitonic peaks present, both of which are associated with  $n = 1$  transitions. Why are there two? There are, in fact, two major kinds of holes: heavy holes and light holes. I won't go into this but yes some holes are heavier than others; don't worry about it.

Now we can finally get to modulation. Just like before, an applied electric field will have an effect on the wavefunctions and thereby the spectrum. Two main effects occur: redshifting and transparency. As a field is applied, the wavefunctions of  $n = 1$  solutions get smushed into the corners and receed from one another, decreasing absorption. The energy gaps between the  $n = 1$  solutions also decrease, redshifting the spectrum.



Figure 12: Wavefunction simulations of 3 QWs of  $In_{0.47}Ga_{0.53}Al_{0.50}P_{0.50}/InP$  with no field (left) and a 5  $V \mu m^{-1}$  field (right)

In the diagram above, see how the field  $n = 1$  solutions are skewed to the left in the conduction band, but skewed to the right in the valence band. This decreases the overlap integral. It's not easy to see but the gaps between  $n = 1$  solutions are also a bit smaller, giving the desired redshift effect.

The below graph is a simulation of the spectrum changes due to QCSE for the wavefunctions shown above. The bottom axis is wavelength rather than energy so you may want to read it backwards to see what it looks like for photon energy. The peak moves down and modulates the absorption around 1600 nm. Finally we have the principal of operation for QCSE EAMs



Figure 13: Absorption simulations of 3 QWs of  $In_{0.47}Ga_{0.53}Al_{0.50}P_{0.50}/InP$  with no field and a 10  $V \mu m^{-1}$  field

# <span id="page-10-0"></span>4 Epilogue & More on quasi-particles

I'm certain that there are mistakes in this report, primarily in hidden assumptions. Hopefully though the undergraduate reader could come away with at least a conceptual idea of QCSE EAMs. I've been personally tripped up over and over on this jargon problem, namely because I've never been formally taught any photonics/optoelectronics/condensed matter/etc.

I have some final thoughts I'd like to share on the nature of quasi-particles. Note again that I am neither a philosopher nor a condensed matter physicist. The fundamental electron is a different particle than the electron quasi-particle; to distinguish between the two, I'm going to refer to the fundamental particles in bold (electron) and quasi-particles in italics (electron).

Personally, I found that many of the people working in photonics either didn't know they were working with *electrons* or they didn't care for the difference between them and electrons. This struck me as strange because when people discuss the exciton they are very careful with their words, as though it's taboo to even call it a particle. This is even more obvious in the *biexciton*: a triply derived quasi-particle *(biexcitons* are derived from excitons which are derived from electrons and holes, which are derived from electrons). In my mind, there is very little conceptual difference between the *electron* and the exciton when it comes to how realised they are as particles. Either they are both valid or neither are.

I also find it interesting that this kind of doubt isn't thrown at neutrons/protons. They're clearly both derived particles, emergent phenomena, but they are not painted with the label of "quasi-particle". Operating on the wikipedia definition, this is because quasi-particles cannot be isolated; that is they can only exist as emergent condensed phenomena within solids. To me, personally, this bar seems a little silly, and seems to beg the question.

It's my belief that the actual explanation is much simpler: excitons are rare, short-lived and generally not very impactful, so they're not really particles. Chemists wouldn't welcome the announcement of protons as quasi-particles, because they're so useful and apparent. To think of a proton as three muons is usually incredibly pedantic.